Precalculus Unit 3: 3.2 Homework
Rational Function Graphs

For each of the following rational functions, find the domain, the vertical asymptote(s) / hole(s), the horizontal asymptote / slant asymptote, the x-intercept(s), the y-intercept, and draw a sketch. Provide work to support your answer.

1. \( f(x) = \frac{2x+5}{x+1} \)

   Domain: \( \mathbb{R}, x \neq -1 \)

   Vertical Asymptote(s):
   \( x = -1 \)

   Hole:
   None

   x-intercept:
   \( \frac{2x+5}{x+1} = 0 \)
   \( 2x+5 = 0 \)
   \( 2x = -5 \)
   \( x = \frac{-5}{2} \)
   \( \left( -\frac{5}{2}, 0 \right) \)

   y-intercept:
   \( f(0) = \frac{2(0)+5}{0+1} = 5 \)

2. \( f(x) = \frac{2x}{x^2+x-2} = \frac{2x}{(x+2)(x-1)} \)

   Domain: \( \mathbb{R}, x \neq -2, x \neq 1 \)

   Vertical Asymptote(s):
   \( x = -2 \)
   \( x = 1 \)

   Hole:
   None

   x-intercept:
   \( \frac{2x}{(x+2)(x-1)} = 0 \)
   \( 2x = 0 \)
   \( x = 0 \)
   \( (0, 0) \)

   y-intercept:
   \( f(0) = \frac{2(0)}{0^2+0-2} = 0 \)
   \( (0, 0) \)

   Horizontal Asymptote / Slant Asymptote:
   \( \frac{\text{Deg} = 1}{\text{Deg} = 2} = y = 0 \)
3. \( f(x) = \frac{x^2 + 3x}{x^2 + x - 6} = \frac{x(x+3)}{(x+3)(x-2)} \)

**Domain:** \( \mathbb{R}, x \neq -3, x \neq 2 \)

**x-intercept:** \( \frac{x}{x-2} = 0 \)  
\( x = 0 \)  
\( (0,0) \)

**Vertical Asymptote(s):** \( x = 2 \)

**Hole:** \( (-3, \frac{-3}{5}) \)  
\( = (-3, \frac{3}{5}) \)

**y-intercept:**  
\( f(0) = \frac{0^2 + 3(0)}{0^2 + 0 - 6} = \frac{0}{-6} = 0 \)  
\( (0,0) \)

**Horizontal Asymptote / Slant Asymptote:** \( \frac{\text{Deg}}{\text{Deg}} = \frac{2}{2} = 1 \)  
\( y = 1 \)

4. \( f(x) = \frac{2x^2 - 5x + 5}{x - 2} \)

**Domain:** \( \mathbb{R}, x \neq 2 \)

**x-intercept:** \( \frac{2x^2 - 5x + 5}{x - 2} = 0 \)
\( x = \frac{5 \pm \sqrt{25 - 4(2)(5)}}{2(2)} \)
\( x = \frac{5 \pm \sqrt{-15}}{4} = \frac{5 \pm i\sqrt{15}}{4} \)  
No x-intercepts

**Hole:** None

**Horizontal Asymptote / Slant Asymptote:** \( \frac{\text{Deg}}{\text{Deg}} = \frac{2}{1} = 2 \)  
\( y = \frac{2x - 1}{2x^2 - 4x - x + 5} \)

**y-intercept:**  
\( f(0) = \frac{2(0)^2 - 5(0) + 5}{0 - 2} = \frac{5}{-2} = -\frac{5}{2} \)  
\( (0, -\frac{5}{2}) \)
5. Write an example of a rational function that has a vertical asymptote at \( x = 4 \), a hole at \((3, -7)\), and a horizontal asymptote at \( y = 2 \).

\[
 f(x) = \frac{(2x+1)(x-3)}{(x-4)(x-3)} = \frac{2x^2 - 5x - 3}{x^2 - 7x + 12} = f(x)
\]

6. The concentration \( C \) of a chemical in the bloodstream \( t \) hours after injection into muscle tissue is given by \( C = \frac{3t^2 + t}{t^3 + 50}, \ t \geq 0 \).

a.) Determine the horizontal asymptote and interpret its meaning in the context of the problem.

\[
\frac{\text{Deg}= 2}{\text{Deg}= 3} \quad [y = 0] \quad \text{Concentration cannot } \neq 0 \text{ or be negative so there will always be trace amounts.}
\]

b.) Graph the function on a graphing utility and approximate the time when the concentration is the greatest.

\[4.5 \text{ hours} \quad 4.486\]

c.) Use the graphing utility to determine when the concentration is less than 0.345.

\[\text{before 2.645 hours and after 8.322 hours}\]