

**Precalculus – Units 7 and 8 Formulas**  
**Sum and Difference, Double Angle, and Half-Angle Formulas**  
**Product-to-Sum and Sum-to-Product**

**The Sum and Difference Formulas:**

1.  $\sin(A+B) = \sin A \cos B + \cos A \sin B$

2.  $\sin(A-B) = \sin A \cos B - \cos A \sin B$

3.  $\cos(A+B) = \cos A \cos B - \sin A \sin B$

4.  $\cos(A-B) = \cos A \cos B + \sin A \sin B$

5.  $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

6.  $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

**The Double Angle Formulas:**

1.  $\sin 2\theta = 2 \sin \theta \cos \theta$

2.  $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$  or  $\cos 2\theta = 1 - 2 \sin^2 \theta$  or  $\cos 2\theta = 2 \cos^2 \theta - 1$

3.  $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

**Alternate Double Angle Formulas (Power Reducing):**

1.  $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$

2.  $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$

3.  $\tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$

**The Half-Angle Formulas:**

1.  $\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$

2.  $\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$

3.  $\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$

### **Product-to-Sum Formulas:**

$$1. \sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$2. \cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$3. \sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

### **Sum-to Product Formulas:**

$$1. \sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}$$

$$2. \sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cdot \cos \frac{\alpha + \beta}{2}$$

$$3. \cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}$$

$$4. \cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \cdot \sin \frac{\alpha - \beta}{2}$$