Name $\underline{\text{Key}}$

Precalculus Unit 8: Test Review

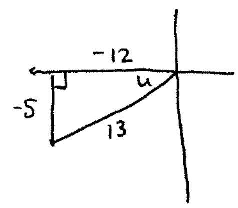
Reminder: When solving equations make sure that you look to see whether a general solution or a solution in a specific interval is asked for.

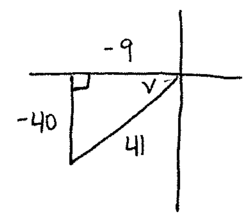
1. Given that $\sin u = \frac{-5}{13}$ and $\cos v = \frac{-9}{41}$ with both u and v in Quadrant III, find the exact value of $\cos(u-v)$.

$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

$$= \left(\frac{-12}{13}\right)\left(\frac{-9}{41}\right) + \left(\frac{-5}{13}\right)\left(\frac{-40}{41}\right)$$

$$= \frac{108}{533} + \frac{200}{533} = \boxed{\frac{308}{533}}$$





Solve the following equations:

2. $2\cos x - \sqrt{3} = 0$ Give solutions on the interval $[0, 2\pi)$.

$$\cos x = \frac{\sqrt{3}}{2}$$

$$\boxed{x = \frac{\pi}{6}, \frac{11\pi}{6}}$$

3. $\csc x + 2 = 0$ Give solutions on the interval $[0, 2\pi)$.

$$\csc x = -2$$

$$\sin x = \frac{-1}{2}$$

$$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$$

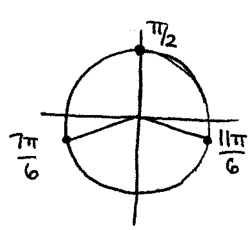
4. $\cos(2x) + \sin x = 0$ Give solutions on the interval $[0, 2\pi)$.

$$1 - 2\sin^2 x + \sin x = 0$$

$$2\sin^2 x - \sin x - 1 = 0$$

$$(2\sin x + 1)(\sin x - 1) = 0$$

$$\sin x = \frac{-1}{2} \qquad \sin x = 1$$



$$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6} \quad x = \frac{\pi}{2}}$$ ← only need this

$$\boxed{x = \frac{\pi}{2} + \frac{2\pi}{3}n, n \in \mathbb{Z}}$$

5. $3\cot^2 x - 1 = 0$ Give the general solution.

$\cot^2 x = \frac{1}{3}$

$\sqrt{\tan^2 x} = \sqrt{3}$

$\tan x = \pm\sqrt{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$$\boxed{\begin{array}{l} x = \frac{\pi}{3} + \pi n, n \in \mathbb{Z} \\ x = \frac{2\pi}{3} + \pi n, n \in \mathbb{Z} \end{array}}$$

6. $1 + tan^2\theta - tan^4\theta = 1$ Give the general solution.

$\tan^2\theta - \tan^4\theta = 0$

$\tan^2\theta(1 - \tan^2\theta) = 0$

$\tan^2\theta = 0 \qquad \tan^2\theta = 1$

$\tan\theta = 0 \qquad \tan\theta = \pm 1$

$\theta = 0, \pi \qquad \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\theta = 0 + \pi n, n \in \mathbb{Z}$

$\boxed{\theta = \pi n, n \in \mathbb{Z}}$

$\boxed{\theta = \frac{\pi}{4} + \frac{\pi}{2} n, n \in \mathbb{Z}}$

7. $\cos\left(x + \frac{\pi}{4}\right) - \cos\left(x - \frac{\pi}{4}\right) = 1$ Give the general solution.

$\left(\cos x \cos\frac{\pi}{4} + \sin x \sin\frac{\pi}{4}\right) - \left(\cos x \cos\frac{\pi}{4} + \sin x \sin\frac{\pi}{4}\right) = 1$

$\frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = 1$

$-\sqrt{2}\sin x = 1$

$\sin x = \frac{-\sqrt{2}}{2}$

$x = \frac{5\pi}{4}, \frac{7\pi}{4}$

$$\boxed{\begin{array}{l} x = \frac{5\pi}{4} + 2\pi n, n \in \mathbb{Z} \\ x = \frac{7\pi}{4} + 2\pi n, n \in \mathbb{Z} \end{array}}$$

8. $8sin^3x - 4sin^2x - 6\sin x + 3 = 0$ Give the general solution. Hint: Factor by grouping.

$4\sin^2 x(2\sin x - 1) - 3(2\sin x - 1) = 0$

$(4\sin^2 x - 3)(2\sin x - 1) = 0$

$\sin^2 x = \frac{3}{4} \qquad \sin x = \frac{1}{2}$

$\sin x = \pm\frac{\sqrt{3}}{2} \qquad x = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$$\boxed{\begin{array}{l} x = \frac{\pi}{6} + 2\pi n, n \in \mathbb{Z} \\ x = \frac{5\pi}{6} + 2\pi n, n \in \mathbb{Z} \\ x = \frac{\pi}{3} + \pi n, n \in \mathbb{Z} \\ x = \frac{2\pi}{3} + \pi n, n \in \mathbb{Z} \end{array}}$$