Name Key

Precalculus Unit 4: 4.1-4.3 Review

Exponential and Logarithmic Functions

(10 points)

1. What is the base of a natural logarithm? What is the base of a common logarithm? (.5)

   Natural log ⇒ e

   Common log ⇒ 10

2. Since 1950, the growth in world population in millions closely fits the exponential function defined by $A(t) = 2600e^{0.018t}$ where t is the number of years since 1950. (1)

   a. Use the function to approximate the population in 1990 (The actual population in 1990 was about 5,320 million).

   $A = 2600e^{.018(40)} = 5341.526$ million

   5,341,526,348

   b. Estimate the population in 2020.

   $A = 2600e^{.018(70)}$

   $A = 9166.096$ million 9,166,095,867

3. $1500 is invested at a rate of 8% compounded quarterly. What is the balance at the end of five years? (.5)

   $A = 1500\left(1 + \frac{.08}{4}\right)^{4(5)} =$ $2228.92

4. $3500 is invested at a rate of 4.5% compounded continuously. What is the balance at the end of 15 years? (.5)

   $A = 3500e^{.045(15)} =$ $6874.12

5. Evaluate the following logarithms without using a calculator: (2)

   a. $\log_5 \frac{1}{125}$ −3

   b. $\log_{25} 5$ $\frac{1}{2}$

   c. $\log_8 8^{2x+1}$ 2x+1

   d. $\log_9 27$ $\log_{3^2} 3^3$ $(3^2)^x = 3^3$ $x = \frac{3}{2}$

   e. $\log_7 1$ 0

   f. $\log_a \frac{1}{a}$ −1

   g. $\ln e^{1-x}$ 1−x

   h. $\log_a a^3$ 3

6. Using the graph of $y = 2^x$ as the base function, explain the transformations in the following graphs. (1)

a. $y = 3 - 2^{x-1}$

up 3
flipped over x-axis
right 1

b. $y = 2^{-x} - 8$

flipped over y-axis
down 8

7. Complete each of the following properties: (1)

a. $\log_a x =$ $\frac{\log x}{\log a}$

b. $\log_a m - \log_a n =$ $\log_a \left(\frac{m}{n}\right)$

c. $\log_a(m \cdot n) =$ $\log_a m + \log_a n$

d. $r \log_a m =$ $\log_a m^r$

8. Write $3^5 = 243$ in logarithmic form. (.5)

$\log_3 243 = 5$

9. Write $\log_3 81 = x$ in exponential form. (.5)

$3^x = 81$

10. Evaluate $\log_3 10$ using change of base. (.5)

2.096

11. Expand to a sum or difference of logarithms. (1)

a. $\log \sqrt{\frac{a^2 b}{c}}$

$\frac{1}{2}\log_a a^2 + \frac{1}{2}\log_a b - \frac{1}{2}\log c$

$\log a + \frac{1}{2}\log b - \frac{1}{2}\log c$

b. $\log_b \frac{x^3 w^3}{z\sqrt{y}}$

$3\log_b x + 3\log_b w - 2\log_b z - \frac{1}{2}\log_b y$

12. Simplify each expression into a single logarithmic quantity. (1)

a. $\frac{1}{4}\log_b 16 - 2\log_b 5 + \log_b 7$

$\log_b\left(\frac{16^{1/4} \cdot 7}{5^2}\right) = \log_b\left(\frac{14}{25}\right)$

b. $\frac{1}{2}\log_y(p^3 q^4) - \frac{2}{3}\log_y(p^4 q^3)$

$\log_y\left(\frac{p^{3/2} q^2}{p^{8/3} q^2}\right) = \log_y\left(p^{-7/6}\right)$