Name Key

Precalculus Unit 12: Review
12.1-12.5

Use the formulas below as needed, but show work!

Finite Arithmetic Series: $S_n = \frac{n}{2}(a_1 + a_n)$

Finite Geometric Series: $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$

Infinite Geometric Series $\sum_{n=1}^{\infty}\left(a_1 r^{n-1}\right) = a_1\left(\frac{1}{1-r}\right)$, when $|r|<1$

Problems:

For problems 1-3, write the first **three** terms of the following sequences (find a_1, a_2, and a_3): (3 points each):

1. $a_n = \frac{3n}{(n-1)!}$

$a_1 = \frac{3(1)}{(1-1)!} = \frac{3}{0!} = \frac{3}{1} = 3$ $\{3, 6, 4.5\}$

$a_2 = \frac{3(2)}{(2-1)!} = \frac{6}{1!} = \frac{6}{1} = 6$ $a_3 = \frac{3(3)}{(3-1)!} = \frac{9}{2!} = \frac{9}{2} = 4.5$

2. $a_{n+1} = -3a_n + 2 \quad a_1 = 5$

$a_1 = 5$ $\{5, -13, 41\}$

$a_2 = -3(5) + 2 = -13$

$a_3 = -3(-13) + 2 = 41$

3. A geometric sequence with $a_1 = -6$ and $r = 3$

$a_1 = -6$ $\{-6, -18, -54\}$

$a_2 = -6 \cdot 3 = -18$

$a_3 = -18 \cdot 3 = -54$

4. Find a_{50} for the arithmetic sequence 23, 21, 19, 17, ... (4 points).

0 1 2
25 -2 -2 -2 → write a formula for the sequence

$a_{50} = -2(50) + 25$ $a_n = -2n + 25$

$\boxed{a_{50} = -75}$

Identify each of the following sequences as arithmetic, geometric, or neither **and** write the formula for the nth term (4 points each):

5. $\frac{4}{3}, \frac{5}{9}, \frac{6}{27}, \frac{7}{81}, \frac{8}{243}, \ldots$

→ there is not a common # being added each time or a common # being multiplied so this is $\boxed{\text{neither}}$

To write the formula, with fractions, it is sometimes easier to cosider the numerator separate from the denominator

numerator: 4, 5, 6, 7, 8, ... this part is arithmetic
+1 +1
$a_n = 1n + 3$

denominator: 3, 9, 27, 81, 243 ... this is geometric
so $a_n = 3(3)^{n-1}$
-or- 3^n

$\boxed{a_n = \frac{n+3}{3(3)^{n-1}} = \frac{n+3}{3^n}}$

6. ~~-8~~ -2, 4, 10, 16, ~~23~~ 22, ...

0 1 2
+6 +6 +6 +6

this is arithmetic

$\boxed{a_n = 6n - 8}$

7. $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$ this sequence is $\boxed{\text{geometric}}$ $a_n = a_1 \cdot r^{n-1}$

$\times\frac{1}{2}\ \times\frac{1}{2}\ \times\frac{1}{2}$

$a_n = 2 \cdot \left(\frac{1}{2}\right)^{n-1}$

Evaluate the following sums. SHOW WORK! (3 points each):

this is arithmetic

8. $\sum_{i=3}^{8}(3i+4) = 13 + 16 + \ldots + 25 + 28$ 41

$S = (41)\left(\frac{6}{2}\right)$

$= 41(3)$

$\boxed{= 123}$

$a_3 = 3(3)+4 = 13$

$a_8 = 3(8)+4 = 28$

terms 3-8 is 6 terms = 3 pairs

9. Find the sum of the first 400 positive integers. (Hint: This is an arithmetic sequence.)

$1 + 2 + 3 + 4 + \cdots + 398 + 399 + 400$

401

$1 + 2 + \ldots + 399 + 400$ $S = (401)\left(\frac{400}{2}\right)$

$= (401)(200) = 80{,}200$

10. $\sum_{n=1}^{100} 6\left(\frac{2}{3}\right)^{n-1}$

$S = a_1\left(\frac{1-r^n}{1-r}\right) = 6\left(\frac{1-(2/3)^{100}}{1-2/3}\right) = \boxed{18}$

↳ geometric (finite)

$a_1 = 6(2/3)^{1-1} = 6$ $r = 2/3$ $n = 100$

11. $\sum_{n=1}^{\infty} 25(0.03)^n$

$S_\infty = \frac{a_1}{1-r} = \frac{.75}{1-.03} = \boxed{\frac{75}{97}}$

↳ geometric (infinite)

$a_1 = 25(.03)^1 = .75$ $r = .03$

12. $\sum_{n=5}^{30} \frac{2^n}{5}$

↳ geometric (finite)

$S = a_1\left(\frac{1-r^n}{1-r}\right) = \frac{32}{5}\left(\frac{1-2^{26}}{1-2}\right)$

$= 429{,}496{,}723.2$

this is the first term being added

$a_5 = \frac{2^5}{5} = \frac{32}{5}$

$r = 2$

$n = 26$ terms 5-30 is 26 terms

13. Use mathematical induction to prove that $2+7+12+17+\ldots+(5n-3)=\frac{5}{2}n^2-\frac{1}{2}n$.

a. What is the formula for a_n? (1 point)

$a_n = 5n - 3$

b. What is the formula for S_n? (1 point)

$S_n = \frac{5}{2}n^2 - \frac{1}{2}n$

c. Show that it is true for $n=1$. (2 points)

$a_1 = 5(1) - 3 = 2$

$S_1 = \frac{5}{2}(1)^2 - \frac{1}{2}(1) = 2$

$\therefore a_1 = S_1$ and S_n is true for $n = 1$

d. Assume that it is true for $n=k$. (2 points)

$S_k = \frac{5}{2}k^2 - \frac{1}{2}k$

e. Show that it is true for $n=k+1$ (4 points)

$S_{k+1} = \frac{5}{2}(k+1)^2 - \frac{1}{2}(k+1)$

$= \frac{5}{2}(k^2+2k+1) - \frac{1}{2}k - \frac{1}{2}$

$= 5/2\, k^2 + 5k + 5/2 - \frac{1}{2}k - \frac{1}{2}$

$= \frac{5}{2}k^2 + \frac{9}{2}k + 2$

$S_{k+1} = S_k + a_{k+1}$

$= \left(\frac{5}{2}k^2 - \frac{1}{2}k\right) + \left(5(k+1) - 3\right)$

$= \frac{5}{2}k^2 - \frac{1}{2}k + 5k + 5 - 3$

$= \frac{5}{2}k^2 + \frac{9}{2}k + 2$

$=$

$\therefore S_k \Rightarrow S_{k+1}$ is true

$\therefore S_n$ is true for all $n \in \mathbb{Z}^+$

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\ 1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \end{array}$$

14. Expand $(x-2y)^6$ and simplify. SHOW WORK! (5 points).

$15(4) \quad 20(-8) \quad 15(16) \quad 6(-32)$

$$=1(x)^6(\cancel{-2y})^0+6(x)^5(-2y)^1+15(x)^4(-2y)^2+20(x)^3(-2y)^3+15(x)^2(-2y)^4+6(x)^1(-2y)^5+1(-2y)^6$$

$$= x^6-12x^5y+60x^4y^2-160x^3y^3+240x^2y^4-192xy^5+64y^6$$

15. Expand $(3x-2i)^4$ and simplify (use $i^2=-1$, etc...). SHOW WORK! (5 points)

$4(27)(-2) \quad 6(9)(4) \quad 4(3)(-8)$

$$1(3x)^4+4(3x)^3(-2i)^1+6(3x)^2(-2i)^2+4(3x)^1(-2i)^3+1(-2i)^4$$

$$= 81x^4-216x^3i+216x^2\underbrace{i^2}_{(-1)}-96x\underbrace{i^3}_{(-i)}+16\underbrace{i^4}_{(1)}$$

$$= 81x^4-216x^3i-216x^2+96xi+16$$

$i=i$

$i^2=-1$

$i^3=-i$

$i^4=1$