

Precalculus Unit 12: Review

12.1-12.5

Use the formulas below as needed, but show work!

Finite Arithmetic Series:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Finite Geometric Series:

$$S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$$

Infinite Geometric Series

$$\sum_{n=1}^{\infty} (a_1 r^{n-1}) = a_1 \left(\frac{1}{1-r} \right), \text{ when } |r| < 1$$

Problems:

For problems 1-3, write the first **three** terms of the following sequences (find a_1 , a_2 , and a_3):
(3 points each):

1. $a_n = \frac{3n}{(n-1)!}$

2. $a_{n+1} = -3a_n + 2$ $a_1 = 5$

3. A geometric sequence with $a_1 = -6$ and $r = 3$

4. Find a_{50} for the arithmetic sequence 23, 21, 19, 17, ... (4 points).

Identify each of the following sequences as arithmetic, geometric, or neither **and** write the formula for the nth term (4 points each):

5. $\frac{4}{3}, \frac{5}{9}, \frac{6}{27}, \frac{7}{81}, \frac{8}{243}, \dots$

6. -2, 4, 10, 16, 24, ...

7. $2, 1, \frac{1}{2}, \frac{1}{4}, \dots$

Evaluate the following sums. SHOW WORK! (3 points each):

8. $\sum_{i=3}^8 (3i + 4)$

9. Find the sum of the first 400 positive integers. (Hint: This is an arithmetic sequence.)

$$1 + 2 + 3 + 4 + \dots + 398 + 399 + 400$$

10. $\sum_{n=1}^{100} 6\left(\frac{2}{3}\right)^{n-1}$

11. $\sum_{n=1}^{\infty} 25(0.03)^n$

12. $\sum_{n=5}^{30} \frac{2^n}{5}$

13. Use mathematical induction to prove that $2 + 7 + 12 + 17 + \dots + (5n - 3) = \frac{5}{2}n^2 - \frac{1}{2}n$.

a. What is the formula for a_n ? (1 point)

b. What is the formula for S_n ? (1 point)

c. Show that it is true for $n = 1$. (2 points)

d. Assume that it is true for $n = k$. (2 points)

$$S_k =$$

e. Show that it is true for $n = k + 1$ (4 points)

$$S_{k+1} =$$

$$S_{k+1} = S_k + a_{k+1}$$

14. Expand $(x - 2y)^6$ and simplify. SHOW WORK! (5 points).

15. Expand $(3x - 2i)^4$ and simplify (use $i^2 = -1$, etc...). SHOW WORK! (5 points)