Precalculus: Semester I Review

Topics:

Chapter 1: Functions and Their Graphs

- Lines: Slopes, writing equations, and graphs
- Functions: Definition, evaluation, and graphs (including graph shifts)
- Domain and Range of a Function
- Combinations and Compositions of Functions
- Inverse Functions

Chapter 2: Polynomial and Rational Functions

- Quadratic Functions
- Zeros of Polynomial Functions
- Complex Numbers
- Rational Functions and their graphs

Chapter 3: Exponential Functions and Logarithmic Functions

- Exponential Functions and Their Graphs
- Logarithmic Functions and Their Graphs
- Solving Exponential and Logarithmic Equations

Chapter 4: Trigonometric Functions

- Radian and Degree Measure
- Using the Unit Circle
- Right Triangle Trig
- Graphs of Trig Functions
- Inverse Trig Functions

The following are some examples to help prepare you for the semester exam. These are by no means all inclusive, which means there probably will be other types of questions on the test. I would also suggest going back through your old tests to look at problems. I have included the solutions to these problems at the end of this review so that you can check your work. I will be available before and after school if you have any questions! Enjoy!

Precalculus: Semester I Review

1. Find the slope passing through the pair of points.

   $(-2, 8), (8, 8)$

   (a) 0 (b) $\frac{8}{5}$ (c) $\frac{5}{3}$ (d) Undefined

2. Find the slope-intercept form of the equation of the line that passes through the given point and has the indicated slope.

   $(0, -4)$, $m = -2$

   (a) $y = -\frac{1}{2}x + 4$ (b) $x = -2y - 4$ (c) $y = -2x + 4$ (d) $y = -2x - 4$

3. Find the slope-intercept form of the equation of the line through the point $(-7, -2)$, parallel to the line $5x + 4y = 4$.

   (a) $y = -\frac{4}{5}x + \frac{4}{43}$ (b) $y = -\frac{5}{4}x + \frac{4}{43}$ (c) $y = -\frac{5}{4}x - \frac{43}{4}$ (d) $y = \frac{5}{4}x - \frac{43}{4}$

4. Determine whether the lines L_1 and L_2 passing through the pair of points are parallel, perpendicular, or neither.

   $L_1: (1, -1), (7, 1)$

   $L_2: (3, -10), (9, -7)$

   (a) Parallel (b) Perpendicular (c) Neither

5. Find a set of ordered pairs (x, y) that represents y as a function of x.

   (a) $\{(-1, -2), (-2, -1), (0, 0)\}$ (b) $\{(-1, -2), (-2, -3), (-1, 0)\}$

   (c) $\{(-1, -2), (-3, 0), (-3, -1), (0, -3)\}$ (d) $\{-1, -2, -3, 0\}$

6. Evaluate the function at the specified value(s) of the independent variable and simplify.

   $g(x) = \frac{x^2 - 3}{3x}$; $g(n-2)$

   (a) $\frac{n^2 - 5}{3n - 6}$ (b) $\frac{n^2 - 4n + 1}{3n - 6}$ (c) $\frac{n^2 - 3}{3n} - 2$ (d) $\frac{n^2 - 4n + 1}{3n - 2}$

7. Find the domain of the function.

$$f(x)=\frac{x^2-2x-8}{x^2-2x-35}$$

(a) All real numbers $x \neq 4, -2$

(b) All real numbers $x \neq -4, 2$

(c) All real numbers $x \neq 7, -5$

(d) All real numbers $x \neq -7, 5$

8. A turtle travels at a rate of 0.2 meter per minute. Let t be the number of minutes spent traveling.
a. Find the distance d traveled by the turtle as a function of t.
b. Find the distance traveled by the turtle in 5 minutes.
c. Construct a graph illustrating the distance traveled by the turtle versus time.

(a) a. $d(t)=t+0.2$
b. 0.7 m

(b) a. $d(t)=0.2t+1$
b. 1.1 m

(c) a. $d(t)=0.2t$
b. 1 m

(d) a. $d(t)=0.2d$
b. 0.8 m

9. Find $f(x+h)$, $f(x+h)-f(x)$, and $\dfrac{f(x+h)-f(x)}{h}$ for the function $f(x)=6x$.

(a) $6x+6h,\ h,\ -6h$

(b) $6x-6h,\ h, 6$

(c) $6x-6h,\ 6h,\ -6h$

(d) $6x+6h,\ 6h, 6$

10. Find the domain and the range of the function.

$$f(x)=7|x+5|$$

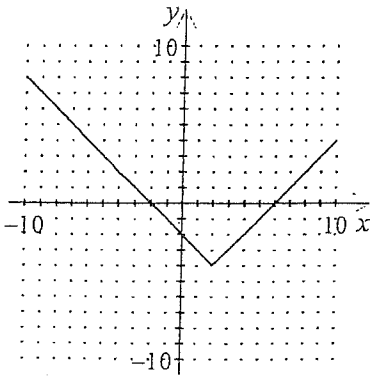
(a) Domain: $[0,\infty)$; Range: $(-\infty,\infty)$

(b) Domain: $(-\infty,\infty)$; Range: $[0,\infty)$

(c) Domain: $(-\infty,\infty)$; Range: $[7,\infty)$

(d) Domain: $[-5,\infty)$; Range: $[0,\infty)$

11. Determine the intervals on which the function is increasing, decreasing, or constant.



$$y=|x-2|-4$$

(a) Increasing on $(2, \infty)$; Decreasing on $(-\infty, 2)$

(b) Increasing on $(-\infty, 2)$; Decreasing on $(2, \infty)$

(c) Increasing on $(4, \infty)$; Decreasing on $(-\infty, 4)$

(d) Increasing on $(-4, \infty)$; Decreasing on $(-\infty, -4)$

12. Use a graphing utility to approximate (to two decimal places) any relative maximum or minimum values of the function.

$y = -x^2 + 4x - 1$

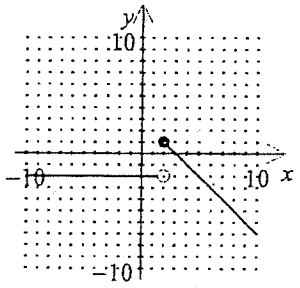
(a) Relative minimum: $(3.00,\ 2.00)$

(b) Relative maximum: $(2.00,\ 3.00)$

(c) Relative maximum: $(3.00,\ 2.00)$

(d) Relative minimum: $(2.00,\ 3.00)$

13. Find the function that represents the graph.



(a) $f(x) = \begin{cases} -2, & x > 2 \\ -x + 3, & x \geq 2 \end{cases}$

(b) $f(x) = \begin{cases} -2, & x > 2 \\ -x + 3, & x \leq 2 \end{cases}$

(c) $f(x) = \begin{cases} -2, & x < 2 \\ -x + 3, & x \leq 2 \end{cases}$

(d) $f(x) = \begin{cases} -2, & x < 2 \\ -x + 3, & x \geq 2 \end{cases}$

14. Find $(f + g)(x)$ for $f(x) = 25 - x^2$ and $g(x) = 5 - x$.

(a) $-x^2 + x + 20$ (b) $-x^2 - x + 30$ (c) $x^3 - 5x^2 - 25x + 125$ (d) $5 + x$

15. If $f(x) = 3 + x$ and $g(x) = x^2 - 2$, find (a) $(g \circ f)(x)$ and (b) $(f \circ g)(x)$.

(a) (a) $x^2 + 1$
(b) $x^2 + x - 1$

(b) (a) $x^2 + x - 1$
(b) $x^2 + x + 6$

(c) (a) $x^2 + 6x + 7$
(b) $x^2 + 1$

(d) (a) $x^2 + x + 1$
(b) $x^2 + 6x + 7$

16. Are the following functions inverses? If not, rewrite the second function so that it is an inverse of the first.

$f(x) = 4x - \frac{2}{3}$

$g(x) = \frac{4}{3}x + \frac{1}{2}$

(a) Yes

(b) No; $g(x) = \frac{1}{4}x + \frac{1}{6}$

(c) No; $g(x) = \frac{3}{4}x - \frac{1}{2}$

(d) No; $g(x) = \frac{1}{3}x - \frac{1}{6}$

17. Find the inverse of the function.

$$f(x)=\frac{-5+4x}{-2+3x}$$

(a) $f^{-1}(x)=\dfrac{3-2x}{4-5x}$ (b) $f^{-1}(x)=\dfrac{2x-5}{3x-4}$ (c) $f^{-1}(x)=\dfrac{4-5x}{3-2x}$ (d) $f^{-1}(x)=\dfrac{3x-2}{4x-5}$

18. Find the equation of a quadratic function whose graph opens upward.

(a) $f(x)=-9x^2+6$ (b) $f(x)=2x^2-10$

(c) $f(x)=-9(-4x-10)^2$ (d) $f(x)=2(6x+6)$

19. The manager of a new company predicted that the company would lose \$2250 its first month, \$1950 its second month, and so on. She further predicted that this trend would continue and that the business would continue to improve by \$300 each month.
a. Find the quadratic equation in standard form that describes the net financial position as a function of months.
b. In which month would the net financial position first become positive?

(a) a. $P(m)=200(m-8)^2-9700$
b. Month 18

(b) a. $P(m)=150(m-8)^2-9600$
b. Month 18

(c) a. $P(m)=150(m-8)^2-9600$
b. Month 17

(d) a. $P(m)=200(m-8)^2-9700$
b. Month 17

20. Use the Intermediate Value Theorem and a graphing utility to find intervals 1 unit in length in which the polynomial function is guaranteed to have a zero. $f(x)=0.1x^3+0.2x^2-0.5x-0.8$

(a) $[-3,-2]$, $[-2,-1]$, $[3,3]$

(b) $[-3,-2]$, $[-2,-1]$, $[2,3]$

(c) $[-3,-1]$, $[-2,-2]$, $[-2,3]$

(d) $[-2,-3]$, $[-1,-2]$, $[3,2]$

21. Use long division or synthetic division to divide.

$$\left(-3x^3-3x+3\right)\div(x-3)$$

(a) $-3x^2-12x+36+\dfrac{111}{x-3}$

(b) $-3x^2-9x-30-\dfrac{87}{x-3}$

(c) $-3x^2-12x-33-\dfrac{99}{x-3}$

(d) $-3x^2-9x+24+\dfrac{72}{x-3}$

22. Factor $4x^3+9x^2-30x-8$ given that $x+4$ is one of its factors.

(a) $(x+4)(x+2)(4x+1)$

(b) $(x+4)(x+2)(4x-1)$

(c) $(x+4)(x-2)(4x+1)$

(d) $(x+4)(x-2)(4x-1)$

DON'T SKIP

22. List the possible rational zeros of f, then use a graphing utility to determine which of those are actual zeros of f.

$f(x) = x^3 + 2x^2 - 21x + 18$

(a) Possible zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 18$
Actual zeros: $-6, 1, 3$

(b) Possible zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$
Actual zeros: $-6, 1, 3$

(c) Possible zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$
Actual zeros: $-1, 6, -3$

(d) Possible zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 18$
Actual zeros: $-1, 6, -3$

DON'T SKIP

23. Use Descartes's Rule of Signs to determine the possible number of positive and negative zeros of the function.

$f(x) = x^6 - 4x^5 - x^4 + x^3 + 2x^2 - 5x + 2$

(a) Two or no positive zeros
Four, two, or no negative zeros

(b) Four, two, or no positive zeros
Three or one negative zeros

(c) Five, three, or one positive zeros
Two or no negative zeros

(d) Four, two, or no positive zeros
Two or no negative zeros

24. Perform the indicated operation and write the result in standard form.

$(-1 - 8i) + (8 + 4i)$

(a) $7 + 4i$ (b) $-9 - 12i$ (c) $24 - 68i$ (d) $7 - 4i$

25. Divide and write the result in standard form. $\dfrac{8+i}{5+6i}$

(a) $-\dfrac{46}{61} + \dfrac{43}{61}i$ (b) $\dfrac{46}{61} + \dfrac{43}{61}i$ (c) $-\dfrac{46}{61} - \dfrac{43}{61}i$ (d) $\dfrac{46}{61} - \dfrac{43}{61}i$

26. Find all zeros of the function.

$f(x) = 4x^2 + 5x + 3$

(a) $\dfrac{-5 \pm \sqrt{23}i}{8}$ (b) $\dfrac{-5 \pm \sqrt{73}i}{8}$ (c) $\dfrac{5 \pm \sqrt{73}i}{8}$ (d) $\dfrac{5 \pm \sqrt{23}i}{8}$

27. Find the horizontal asymptote of the graph of $f(x)=\dfrac{3}{x-9}$.

(a) $x=0$ (b) $y=0$ (c) $y=3$ (d) $x=9$

28. Find the vertical asymptote(s), if any, for $f(x)=\dfrac{5x+7}{x^2+3x-10}$.

(a) $x=-5,\ x=2$

(b) $x=7,\ x=-5$

(c) $x=-5,\ x=2,\ x=7$

(d) No vertical asymptotes

29. Determine if the graph of the rational function has a slant asymptote. If it does, find the equation of the slant asymptote.

$$f(x)=\frac{2x^6-9x^2+2x+16}{x^6-3x-4}$$

(a) $y=2x-3$ (b) $y=2x$ (c) $y=2x+3$ (d) No slant asymptote

30. Steven threw a rock from the top of a steep 69-foot high hill into a pond at the foot of the hill. The chart gives the horizontal distance, x (in feet), the rock traveled from Steven and the height, y (in feet), of the rock above the pond.

Distance, x	15	25	42	56
Height, y	76	78	79	75

Find the quadratic equation for the rock's trajectory from Steven to the pond below.

(a) $y=-0.0182x^2+0.549x+69$ (b) $y=(x+34)^2+79$

(c) $y=-0.0082x^2+0.571x+69$ (d) $y=(x+35)^2+79$

31. If \$2500 is invested in a long-term trust fund with an interest rate of 5% compounded continuously, what is the amount of money in the account after 20 years?

(a) \$6633.24 (b) \$18,472.64 (c) \$6795.70 (d) \$7144.13

32. Evaluate the expression without using a calculator.

$\log_3 9$

(a) 2 (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{2}$ (d) 3

33. Evaluate the logarithm using the change of base formula.

$\log_8 \frac{1}{5}$

(a) −3.347 (b) −0.774 (c) −0.201 (d) −12.876

34. Find the expression that is equivalent to the given logarithmic expression.

$\log_b \sqrt{\frac{77}{81}}$

(a) $\frac{1}{2}(\log_b 77 - \log_b 81)$ (b) $\sqrt{\log_b 77 - \log_b 81}$

(c) $\frac{1}{2}(\log_b 77 + \log_b 81)$ (d) $\log_b \frac{1}{2}(77 - 81)$

35. Use the properties of logarithms to expand the expression.

$\log_b \sqrt[2]{\frac{x^4 y^6}{z^8}}$

(a) $\sqrt[2]{\frac{24(\log_b x)(\log_b y)}{8 \log_b z}}$ (b) $2 \log_b x + 3 \log_b y - 4 \log_b z$

(c) $2 \log_b x + \frac{1}{3} \log_b y - \frac{1}{4} \log_b z$ (d) $\log_b (x + y - z)$

36. Condense the expression to the logarithm of a single quantity.

$2 \log_{10} x - 4 \log_{10} (x+2)$

(a) $8 \log_{10} \frac{x}{x+2}$ (b) $\log_{10} \frac{x^2}{(x+2)^4}$ (c) $\log_{10} \frac{x}{(x+2)^4}$ (d) None of these

37. Solve for x. $\frac{1}{27} = 9^{8x+2}$

(a) $-\frac{7}{16}$ (b) $-\frac{5}{8}$ (c) $-\frac{1}{16}$ (d) $-\frac{5}{16}$

38. Solve the exponential equation algebraically.

$2^x = 9^{x+8}$

(a) −5.843 (b) −11.687 (c) 6.082 (d) −10

39. Solve the exponential equation algebraically.

$$\frac{400}{1+e^{-x}} = 375$$

(a) 2.708 (b) –0.661 (c) 2.621 (d) –0.726

40. Find the value of x.

$$\log_3(x+2) - \log_3 x = 3$$

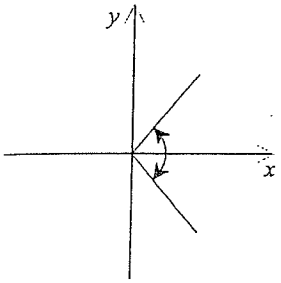
(a) 4 (b) 0.077 (c) 13 (d) 0.250

41. Find the time required for an investment of \$2500 to double if the interest rate of 10% is compounded continuously.

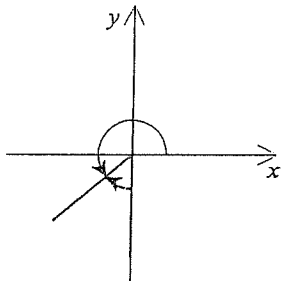
(a) 4.62 years (b) 13.86 years (c) 3.47 years (d) 6.93 years

42. Sketch a pair of coterminal angles.

(a)



(b)



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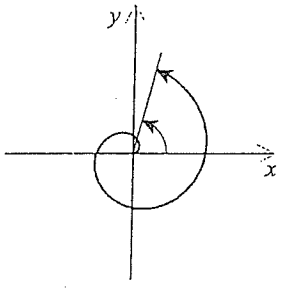
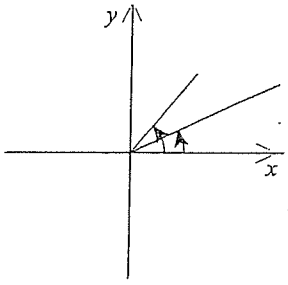


(d)



43. If possible, find the complement of the angle $\theta = \frac{4\pi}{15}$.

(a) $\frac{7\pi}{30}$ (b) $\frac{\pi}{15}$ (c) $\frac{11\pi}{15}$ (d) not possible

44. In which quadrant is the terminal side of the angle θ?

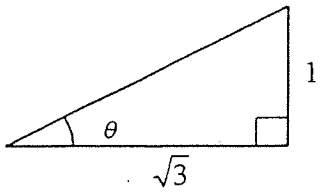
$$\theta = -\frac{9\pi}{10}$$

(a) Quadrant I (b) Quadrant II (c) Quadrant III (d) Quadrant IV

45. A point on the rim of a wheel has a linear speed of 14 cm/s. If the radius of the wheel is 20 cm, what is the angular speed of the wheel in radians per second?

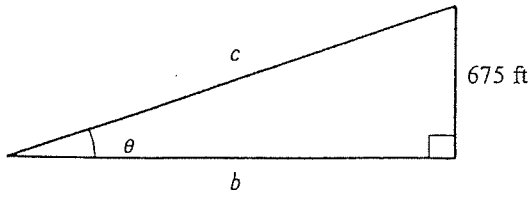
(a) 1.4 rad/s (b) 2.2 rad/s (c) 0.7 rad/s (d) 0.3 rad/s

46. Find the exact value of the sine and cosine functions of the angle θ given in the figure. (Use the Pythagorean Theorem to find the third side of the triangle.)



(a) $\sin\theta = \frac{\sqrt{3}}{2}$, $\cos\theta = \frac{1}{2}$ (b) $\sin\theta = \frac{\sqrt{3}}{3}$, $\cos\theta = \sqrt{3}$ (c) $\sin\theta = \frac{2\sqrt{3}}{3}$, $\cos\theta = \frac{\sqrt{3}}{2}$ (d) $\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$

47. The cable supporting a ski lift rises 3 feet for each 8 feet of horizontal length. The top of the cable is fastened 675 feet above the cable's lowest point. Find the lengths b and c, and find the measure of angle θ.



(a) $b = 253$ ft, $c = 721$ ft, $\theta = 69.4°$ (b) $b = 1800$ ft, $c = 1922$ ft, $\theta = 20.6°$ (c) $b = 1922$ ft, $c = 1800$ ft, $\theta = 22.0°$ (d) $b = 721$ ft, $c = 253$ ft, $\theta = 0.4°$

48. Find the quadrant in which θ lies.

$\tan\theta > 0$ and $\cos\theta < 0$

(a) Quadrant I (b) Quadrant II (c) Quadrant III (d) Quadrant IV

49. Find the reference angle θ'.

$\theta = 3.5$

(a) 2.7832 (b) 3.5 (c) 1.9292 (d) 0.3584

50. Given $\tan\theta = -\frac{12}{35}$ and $\sin\theta > 0$, find $\cos\theta$.

(a) $\cos\theta = \frac{12}{37}$ (b) $\cos\theta = -\frac{35}{37}$ (c) $\cos\theta = -\frac{12}{37}$ (d) $\cos\theta = \frac{35}{37}$

51. Find a function that has the given amplitude and period.
amplitude = 0.5, period = 8π

(a) $y = 0.25\cos\frac{x}{4}$ (b) $y = 0.5\cos\frac{x}{4}$ (c) $y = 0.25\cos 8\pi x$ (d) $y = 0.5\cos\frac{\pi x}{8}$

52. At a distance of 56 feet from the base of a flag pole, the angle of elevation to the top of a flag that is 3.1 feet tall is 25.6°. The angle of elevation to the bottom of the flag is 22.9°. The pole extends 1 foot above the flag. Find the height of the pole.

(a) 26.8 ft (b) 24.8 ft (c) 23.8 ft (d) 27.8 ft

SKIP 53. A hiker travels at 3.9 miles per hour at a heading of S 21° E from a ranger station. After 3.5 hours, how far south and how far east is the hiker from the ranger station?

(a) 4.9 miles south and 12.7 miles east (b) 7.5 miles south and 11.4 miles east

(c) 11.4 miles south and 7.5 miles east (d) 12.7 miles south and 4.9 miles east

SKIP 54. Factor the expression and use the fundamental identities to simplify.
$\cos^2 x \sin^2 x - \cos^2 x$

(a) $-\cos^4 x$ (b) $\cos^2 x$ (c) $\tan^2 x$ (d) 1

SKIP 55. Find an expression that completes the identity.

$$\frac{1+\cos u}{\sin u} + \frac{\sin u}{1+\cos u} =$$

(a) $2+\cos u$ (b) 0 (c) $-\sin u$ (d) $2\csc u$

Answers to Precalculus Semester I Review

Problem #	Answer
1	a
2	d
3	c
4	c
5	a
6	b
7	c
8	c
9	d
10	b
11	a
12	b
13	d
14	b
15	c
16	b
17	b
18	b
19	c
20	b
21	b
22	c
22	b
23	d
24	d
25	d
26	a
27	b

Problem #	Answer
28	a
29	d
30	c
31	c
32	a
33	b
34	a
35	b
36	b
37	a
38	b
39	a
40	b
41	d
42	c
43	a
44	c
45	c
46	d
47	b
48	c
49	d
50	b
51	b
52	d
53	d
54	a
55	d