Precalculus: Chapter 6 Review
Sections 6.1-6.4

SECTION 6.1: The Law of Sines

- If ABC is any oblique triangle with sides *a*, *b*, and *c*, then the Law of Sines says $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$. Remember that this can result in zero, one, or two triangles.

- Examples:

  Use the Law of Sines to find the remaining sides and angles of the triangles given below:

  1. $A=40°$, $B=12°$, $b=100$
  2. $C=150°$, $a=5$, $c=20$

SECTION 6.2: The Law of Cosines

- If ABC is any oblique triangle with sides *a*, *b*, and *c*, then the Law of Cosines says $a^2=b^2+c^2-2bc\cos A$. This can result in zero or one triangle.

- Heron's Formula: Given any triangle with sides of lengths *a*, *b*, and *c*, then the area of the triangle is $Area=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}$.

- Examples:

  1. Use the Law of Cosines to find the remaining sides and angles of the triangles given below:

     a) $a=49$, $b=53$, $c=38$

     b) $C=29°$, $a=100$, $b=300$

  2. Use Heron's Formula to find the area of the triangle: $a=4.1$, $b=6.8$, $c=5.5$

  3. A ship travels 40 miles due east, then adjusts its course $12°$ southward. After traveling 70 miles in that direction, how far is the ship from its point of departure?

SECTION 6.3: Vectors in the Plane

- You should be able to geometrically perform the operations of vector addition (subtraction) and scalar multiplication.

- The component form of the vector with initial point $P = (p_1, p_2)$ and terminal point $Q = (q_1, q_2)$ is $\overrightarrow{PQ} = \langle q_1 - p_1, q_2 - p_2 \rangle$, terminal minus initial.

- The magnitude of $\vec{v} = \langle v_1, v_2 \rangle$ is $\|\vec{v}\| = \sqrt{(v_1)^2 + (v_2)^2}$

- You should be able to perform the operations of scalar multiplication and vector addition in component form.

- A unit vector in the direction of $\vec{v}$ is given by $\vec{u} = \frac{\vec{v}}{\|\vec{v}\|}$.

- The standard unit vectors are $\vec{i} = \langle 1,0 \rangle$ and $\vec{j} = \langle 0,1 \rangle$. $\vec{v} = \langle v_1, v_2 \rangle$ can be written as $\vec{v} = v_1\vec{i} + v_2\vec{j}$.

- A vector $\vec{v}$ with magnitude $\|\vec{v}\|$ and direction angle θ can be written as $\langle \|\vec{v}\| \cos\theta, \|\vec{v}\| \sin\theta \rangle$,

  where $\tan\theta = \frac{y}{x}$.

- Examples:

1. $\vec{w} = 4\vec{u} - 7\vec{v}$, where $\vec{u} = \langle 3,1 \rangle$ and $\vec{v} = \langle -1,2 \rangle$, find $\vec{w}$.

2. Find a unit vector in the direction of $\vec{v} = 5\vec{i} - 3\vec{j}$

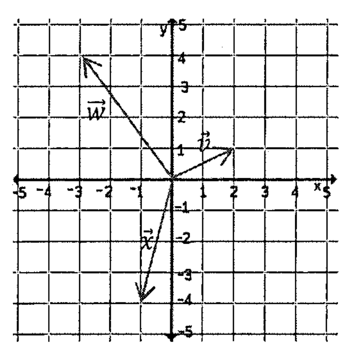
3. $\vec{v}$ is a vector of magnitude 4 making an angle of $30°$. Find $\vec{v}$ in component form.

4. Given the graph at right:

   a) Sketch $2\vec{v}$

   b) Sketch $\vec{w} + \vec{x}$

   c) Sketch $\vec{v} - \vec{x}$



5. Given two vectors with magnitudes of 120 pounds and 80 pounds with an angle of 40° between them, what is the direction and magnitude of the resultant?

SECTION 6.4: Vectors and Dot Products

- The dot product of $\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$ is $\vec{u} \cdot \vec{v} = u_1 \cdot v_1 + u_2 \cdot v_2$. This results in a scalar.

- The formula for finding the angle, θ, between two nonzero vectors $\vec{u}$ and $\vec{v}$ is $\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \cdot \|\vec{v}\|}$.

- The vectors $\vec{u}$ and $\vec{v}$ are orthogonal if $\vec{u} \cdot \vec{v} = 0$

- Know the definition of work: $Work = \vec{F} \bullet \overrightarrow{PQ}$

- Examples:

1. Find the angle between $\vec{v} = \langle 6,5 \rangle$ and $\vec{w} = \langle -3,2 \rangle$.

2. Are the vectors given in question 1, orthogonal, parallel, or neither? How do you know?

ANSWERS:

SECTION 6.1:

1. $C = 128°$, $a = 309.16$, $c = 379.01$

2. $A = 7.18°$, $B = 22.82°$, $b = 15.51$

SECTION 6.2:

1a. $A = 62.63°$, $B = 73.85°$, $C = 43.53°$

1b. $c = 218.00$, $A = 12.85°$, $B = 138.15°$

2. $11.27u^2$

3. approximately 109.44 miles

SECTION 6.3:

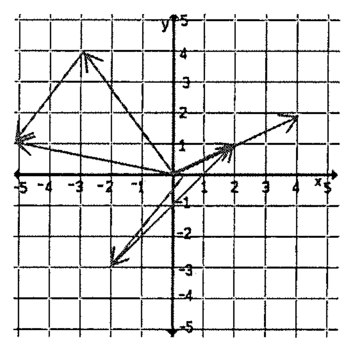
1. $\vec{w} = \langle 19, -10 \rangle$

2. $\frac{\vec{v}}{\|\vec{v}\|} = \frac{5\sqrt{34}}{34}\vec{i} - \frac{3\sqrt{34}}{34}\vec{j} = \langle \frac{5\sqrt{34}}{34}, \frac{-3\sqrt{34}}{34} \rangle$

3. $\langle 2\sqrt{3}, 2 \rangle$

4. See graph.

5. 188.44 pounds



SECTION 6.4:

1. $\theta = 96.12°$

2. Neither – the dot product is not zero and the vectors are not scalar multiples.