Precalculus: Section 1.5
Combinations of Functions

Sum, Difference, Product, and Quotient of Functions

Sum: $(f+g)(x) = f(x) + g(x)$ Difference: $(f-g)(x) = f(x) - g(x)$

Product: $(f \cdot g)(x) = f(x) \cdot g(x)$ Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$

***The domain of a combination of functions is all elements common to the domain of functions *f* and *g*.

*** We looked at an example with polynomials together in class. The following example is one using square roots.

Example: $g(x) = \sqrt{4-x^2}$, $h(x) = \sqrt{x}$

Domain of $g(x) \Rightarrow -2 \leq x \leq 2$; domain of $h(x) \Rightarrow x \geq 0$
(This is so there are no negative values underneath the square roots!)

a) $(g+h)(x) = \sqrt{4-x^2} + \sqrt{x}$ (This is as simplified as this gets – the domain is the overlap of the domains that we found above individually: $0 \leq x \leq 2$

b) $(g-h)(x) = \sqrt{4-x^2} - \sqrt{x}$ (This also cannot be simplified any further – same domain as part a.)

c) $(f \cdot g)(x) = \sqrt{4-x^2} \cdot \sqrt{x} = \sqrt{4x-x^3}$ (You can multiply values that are both under a square root, so this simplifies slightly. The domain is the same as parts a and b.)

d) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{4-x^2}}{\sqrt{x}} = \sqrt{\frac{4-x^2}{x}}$ (Division of roots can be combined under a single root and then reduced if the fraction is reducible. The domain here is similar, but it changes slightly to $0 < x \leq 2$. Notice that the 0 is no longer included because it creates a zero denominator.)

e) $(g+h)(0) = \sqrt{4-0^2} + \sqrt{0} = 2$ (Substitute 0 into $(g+h)(x)$)

*** Take a look at these graphs on the calculator and you will see that the domains fit the results.

Composition of Functions:

$$f\big(g(x)\big) = (f \circ g)(x)$$

Example: $f(x) = \frac{1}{x}$ Domain: All real numbers, $x \neq 0$

$g(x) = x + 5$ Domain: All real numbers

a) $(f \circ g)(x) = \frac{1}{x+5}$ Domain: All real numbers, $x \neq -5$

For $(f \circ g)(x)$, function *g* is substituted into function *f*. That is, function *g* replaces the *x* in function *f*.

b) $(g \circ f)(x) = \frac{1}{x} + 5$ Domain: All real numbers, $x \neq 0$

For $(g \circ f)(x)$, function *f* is substituted into function *g*. That is, function *f* replaces the *x* in function *g*.

Example: $f(x) = x^2 + 3x - 2$ $g(x) = 4x - 5$

a) $(f \circ g)(x) = (4x - 5)^2 + 3(4x - 5) - 2 = 16x^2 - 28x + 8$

b) $(g \circ f)(x) = 4(x^2 + 3x - 2\,) - 5 = 4x^2 + 12x - 13$

Homework: pp. 58-61 #'s 7, 12, 15, 18, 20, 35, 36, 39, 40, 43, 48a, 64, 73, 79