Precalculus Unit 9: 9.4 Notes

Dot Product:

- The dot product of $\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$ is given by $\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2$

  Example: $\vec{u} = \langle 2, -3 \rangle$ and $\vec{v} = \langle -4, -5 \rangle$

  $$\vec{u} \cdot \vec{v} = 2(-4) + (-3)(-5) = -8 + 15 = 7$$

- Properties of the Dot Product

  Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be vectors in the plane or in space and let c be a scalar.

  1. $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
  2. $\vec{0} \cdot \vec{v} = 0$
  3. $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$
  4. $\vec{v} \cdot \vec{v} = ||\vec{v}||^2$
  5. $c(\vec{u} \cdot \vec{v}) = c\vec{u} \cdot \vec{v} = \vec{u} \cdot c\vec{v}$

  *Property 4 gives another way to find magnitude for a vector: $||\vec{v}|| = \sqrt{\vec{v} \cdot \vec{v}}$

Angle Between Two Vectors:

- If θ is the angle between two nonzero vectors $\vec{u}$ and $\vec{v}$, then $\cos\theta = \frac{\vec{u} \cdot \vec{v}}{||\vec{u}||\,||\vec{v}||}$

  Example: Find the angle between $\vec{u} = \langle -3, 1 \rangle$ and $\vec{v} = \langle 5, -2 \rangle$

  $$\vec{u} \cdot \vec{v} = (-3)(5) + (1)(-2) = -15 + -2 = -17$$

  $$||\vec{u}|| = \sqrt{(-3)^2 + 1^2} = \sqrt{10}$$

  $$||\vec{v}|| = \sqrt{5^2 + (-2)^2} = \sqrt{29}$$

  $$\cos\theta = \frac{-17}{\sqrt{10}\sqrt{29}} = \frac{-17}{\sqrt{290}}$$

  $$\theta = 176.63°$$

Orthogonal and Parallel Vectors:

- Orthogonal vectors are perpendicular vectors.

- The vectors $\vec{u}$ and $\vec{v}$ are orthogonal if $\vec{u} \cdot \vec{v} = 0$

  Example: Are $\vec{u} = \langle 3, -9 \rangle$ and $\vec{v} = \langle -3, -1 \rangle$ orthogonal?

  $$\vec{u} \cdot \vec{v} = (3)(-3) + (-9)(-1) = -9 + 9 = 0$$

  The dot product =0 so $\vec{u}$ and $\vec{v}$ are orthogonal.

- Parallel vectors have the same direction which means that if two vectors are parallel, one will be a scalar multiple of the other.

Work:

- The work W done by a constant force $\vec{F}$ as its point of application moves along the vector $\overrightarrow{PQ}$ is given by $W = \vec{F} \cdot \overrightarrow{PQ}$

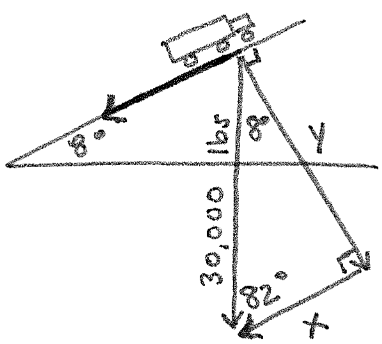
- Example 1: Find the work done in moving a particle from P to Q if the magnitude and direction of the force are given by $\vec{v}$.

  $P = (1,3)$, $Q = (-3,5)$, and $\vec{v}\langle -2,3\rangle$

  $\overrightarrow{PQ} = \langle -4,2\rangle$ (terminal minus initial)

  $\mathrm{W} = \overrightarrow{\mathrm{PQ}} \cdot \vec{\mathrm{v}} = (-4)(-2) + (2)(3) = 8 + 6 = 14$

- Example 2: A truck with a gross weight of 30,000 pounds is parked on a slope of 8°. Assume the only force to overcome is the force of gravity. Find the force required to keep the truck from rolling down the hill and find the force perpendicular to the hill.



$\sin 8° = \dfrac{x}{30{,}000}$

$x = 4175.19$ lbs → rolling down hill

$\cos 8° = \dfrac{y}{30{,}000}$

$y = 29{,}708.04$ lbs. → $\perp$ to hill

- Example 3: A toy wagon is pulled be exerting a force of 20 pounds on a handle that makes a 25° angle with the horizontal. Find the work done in pulling the wagon 40 feet.

  $\vec{F} = \langle 20\cos 25°, 20\sin 25°\rangle = \langle 18.13, 8.45\rangle$

  $\overrightarrow{PQ} = \langle 40,0\rangle$

  $W = \vec{F} \cdot \overrightarrow{PQ} = 725.2$ foot pounds