Name KEY

Precalculus Unit 8 – 8.2 Homework
Solving Trigonometric Equations using Identities

1. Solve: $\cos\left(x+\frac{\pi}{4}\right)-\cos\left(x-\frac{\pi}{4}\right)=1$

$$\left(\cos x\cos\tfrac{\pi}{4}-\sin x\sin\tfrac{\pi}{4}\right)-\left(\cos x\cos\tfrac{\pi}{4}+\sin x\sin\tfrac{\pi}{4}\right)=1$$

$$\tfrac{\sqrt{2}}{2}\cos x-\tfrac{\sqrt{2}}{2}\sin x-\tfrac{\sqrt{2}}{2}\cos x-\tfrac{\sqrt{2}}{2}\sin x=1$$

$$-\sqrt{2}\sin x=1$$

$$\sin x=\frac{-1}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$$

$X=\frac{5\pi}{4},\frac{7\pi}{4}$

$X=\frac{5\pi}{4}+2\pi n, n\in\mathbb{Z}$

$X=\frac{7\pi}{4}+2\pi n, n\in\mathbb{Z}$

2. Solve: $\sin(2x)+\cos x=0$

$$2\sin x\cos x+\cos x=0$$

$$\cos x(2\sin x+1)=0$$

$\cos x=0 \quad \sin x=-\frac{1}{2}$

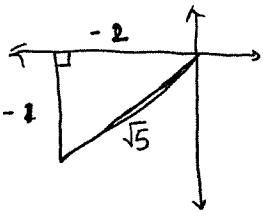
$X=\frac{\pi}{2},\frac{3\pi}{2} \quad X=\frac{7\pi}{6},\frac{11\pi}{6}$

$X=\frac{\pi}{2}+\pi n, n\in\mathbb{Z}$

$X=\frac{7\pi}{6}+2\pi n, n\in\mathbb{Z}$

$X=\frac{11\pi}{6}+2\pi n, n\in\mathbb{Z}$

3. Use the double angle formulas to find the value of $\sin 2u$, $\cos 2u$, *and* $\tan 2u$ given $\tan u=\frac{1}{2}$ and $\pi<u<\frac{3\pi}{2}$.



$$\sin(2u)=2\sin u\cos u$$

$$=2\left(\frac{-1}{\sqrt{5}}\right)\left(\frac{-2}{\sqrt{5}}\right)=\boxed{\frac{4}{5}}$$

$$\cos(2u)=\cos^2u-\sin^2u$$

$$=\left(\frac{-2}{\sqrt{5}}\right)^2-\left(\frac{-1}{\sqrt{5}}\right)^2=\boxed{\frac{3}{5}}$$

$$\tan(2u)=\frac{4/5}{3/5}=\boxed{4/3}$$

4. Solve: $\cos(2x) + 6\sin^2(x) = 4$

$1 - 2\sin^2 x + 6\sin^2 x = 4$

$4\sin^2 x = 3$

$\sin^2 x = \frac{3}{4}$

$\sin x = \pm \frac{\sqrt{3}}{2}$

$x = \pi/3,\ 2\pi/3,\ 4\pi/3,\ 5\pi/3$

$x = \pi/3 + \pi n, n \in \mathbb{Z}$

$x = 2\pi/3 + \pi n, n \in \mathbb{Z}$

5. Solve: $\cos(2x) - \cos(6x) = 0$

$-2\sin\left(\frac{2x+6x}{2}\right)\sin\left(\frac{2x-6x}{2}\right) = 0$

$\sin(4x)\sin(-2x) = 0$

$\sin 4x = 0$

$4x = 0 + \pi n, n \in \mathbb{Z}$

$x = \pi/4 n, n \in \mathbb{Z}$

$-\sin(2x) = 0$

$\sin(2x) = 0$

$2x = 0 + \pi n, n \in \mathbb{Z}$

$x = \frac{\pi}{2} n, n \in \mathbb{Z}$

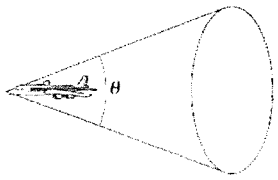
6. Solve: $\sin(2x) + 1 = 0$

$\sin(2x) = -1$

$2x = \frac{3\pi}{2} + 2\pi n, n \in \mathbb{Z}$

$x = \frac{3\pi}{4} + \pi n, n \in \mathbb{Z}$

7. The mach number *M* of an airplane is the ratio of its speed to the speed of sound. When an airplane travels faster than the speed of sound, the sound waves form a cone behind the airplane (see figure). The mach number is related to the apex angle θ of the cone by $\sin\frac{\theta}{2} = \frac{1}{M}$.



a. Find the angle θ that corresponds to a mach number of 1.

$\sin\frac{\theta}{2} = \frac{1}{1}$ $\quad \frac{\theta}{2} = \frac{\pi}{2} \rightarrow$ $\theta = \pi = 180°$

b. Find the angle θ that corresponds to a mach number of 4.5.

$\sin\frac{\theta}{2} = \frac{1}{4.5}$ $\quad \frac{\theta}{2} = 12.84°$ $\quad \theta = 25.68°$

c. The speed of sound is about 760 miles per hour. Determine the speed of an object having a mach number of 1 and the speed of an object with a mach number of 4.5.

Mach 1 = 760 mph

Mach 4.5 = 3420 mph