

PreCalculus Unit 4: Section 4.1 Notes Exponential Functions

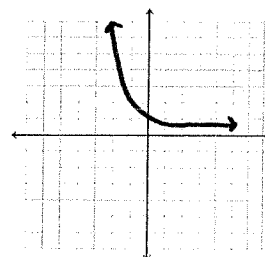
Definition of an Exponential Function: $f(x) = a^x$ where $a > 0, a \neq 1$, and x is any real number.

Let us explore this concept! Graph the function $f(x) = a^x$

1. What happens to the function when $0 < a < 1$? Sketch the graph.

The graph becomes a decreasing graph

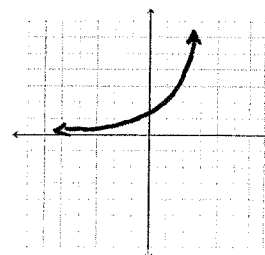
EX: $y = (\frac{1}{2})^x$



2. What happens to the function when $a > 1$? Sketch the graph.

The graph is an increasing graph and gets steeper as the base increases

EX: $y = 2^x$



3. What happens to the function when $a < 0$?

The graph becomes a series of disconnected points and is not an exponential function

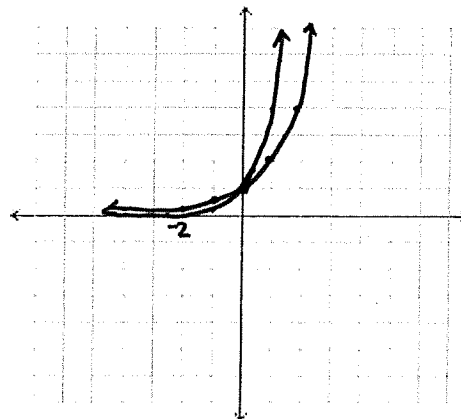
4. What happens to the function when $a = 0$? How about when $a = 1$?

$a = 0$: The graph is a horizontal ray starting at zero and moving in the positive direction

$a = 1$: The graph is a horizontal line at $y = 0$ * Neither of these are exponential.

5. Graph the functions $f(x) = 2^x$ and $g(x) = 4^x$ and create a table of values for each function.

x	-2	-1	0	1	2
2^x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
4^x	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16



Are these functions increasing or decreasing?

Increasing

Which function is increasing or decreasing faster? Explain your reasoning.

4^x increases more quickly when $x > 0$

Identify the domain, range, y-intercept, horizontal asymptote, and whether the function is increasing or decreasing for both functions.

$$f(x) = 2^x$$

a) Domain: \mathbb{R}

b) Range: $y > 0$

c) Increasing or decreasing:

d) Y-intercept: $(0, 1)$

e) Horizontal asymptote: $y = 0$

$$g(x) = 4^x$$

Domain: \mathbb{R}

Range: $y > 0$

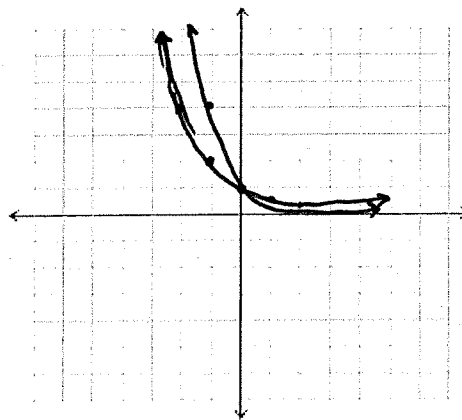
Increasing or decreasing:

Y-intercept: $(0, 1)$

Horizontal Asymptote: $y = 0$

6. Graph the functions $f(x) = 2^{-x}$ and $g(x) = 4^{-x}$ and create a table of values for each function.

x	-2	-1	0	1	2
2^{-x}	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$
4^{-x}	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$



Are these functions ~~increasing~~ or decreasing?

Which function is increasing or decreasing faster? Explain your reasoning.

4^{-x} decreases faster

Identify the domain, range, y-intercept, horizontal asymptote, and whether the function is increasing or decreasing for both functions.

$$f(x) = 2^{-x}$$

a) Domain: \mathbb{R}

b) Range: $y > 0$

c) Increasing or decreasing:

d) Y-intercept: $(0, 1)$

e) Horizontal asymptote: $y = 0$

$$g(x) = 4^{-x}$$

Domain: \mathbb{R}

Range: $y > 0$

Increasing or decreasing:

Y-intercept: $(0, 1)$

Horizontal Asymptote: $y = 0$

Transformations of Graphs of Exponential Functions

1. Graph the functions $f(x) = 3^x$ and $f(x) = 3^{(x+2)}$

What happens to the graph when we add 2 to the exponent?

It moves 2 to the left

a) Does the domain of the function change? If so, how?

No

b) Does the range change? If so, how?

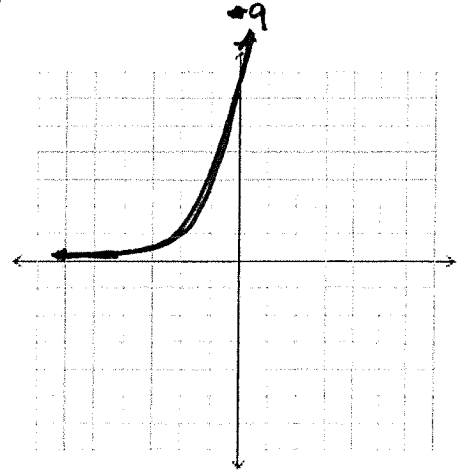
No

c) Does the y-intercept change? If so, how?

Yes $f(0) = 3^{0+2} = 3^2 = 9$ I moves from (0,1) to (0,9)

d) Does the horizontal asymptote change? If so, how?

No



* Notice when you add n to the exponent, the graph of the function shifts n units to the left. Similarly, when you subtract n from the exponent, the graph of the function shifts n units to the right.

2. Graph the functions $f(x) = 3^x$ and $f(x) = 3^x + 2$

What happens to the graph when we add 2 outside of the exponent?

It moves up 2

a) Does the domain of the function change? If so, how?

No

b) Does the range change? If so, how?

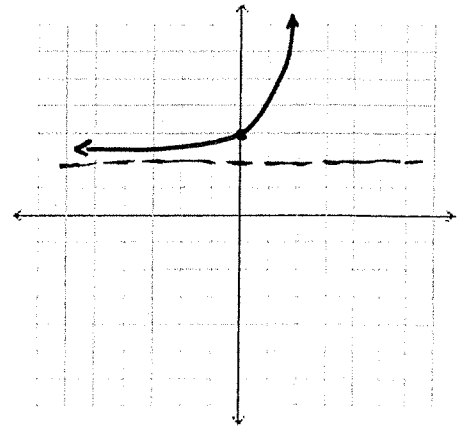
Yes $y > 2$

c) Does the y-intercept change? If so, how?

yes It moves up two, to (0,3)

d) Does the horizontal asymptote change? If so, how?

yes, It moves up 2 to $y=2$



* Notice when you add n outside of the exponent, the graph shifts upwards n units. Similarly, when you subtract n outside of the exponent, the graph shifts downwards n units.

3. Graph the functions $f(x) = 3^x$ and $f(x) = 3^{-x}$

What happens to the graph when we make the exponent negative?

It flips over the y-axis.
(reflects)

a) Does the domain of the function change? If so, how?

No

b) Does the range change? If so, how?

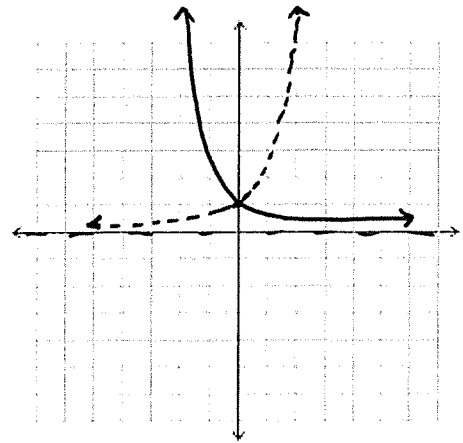
No

c) Does the y-intercept change? If so, how?

No

d) Does the horizontal asymptote change? If so, how?

No



* Notice when the exponent is negative, the graph reflects over the y-axis.

4. Graph the functions $f(x) = 3^x$ and $f(x) = -3^x$

What happens to the graph when we make the constant negative?

It reflects over the x-axis

a) Does the domain of the function change? If so, how?

~~No~~ No

b) Does the range change? If so, how?

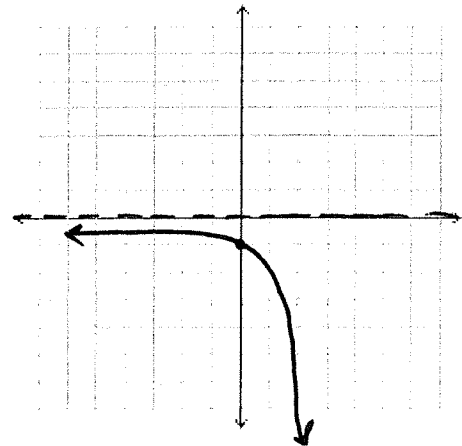
Yes it becomes $y < 0$

c) Does the y-intercept change? If so, how?

yes, it flips to (0, -1)

d) Does the horizontal asymptote change? If so, how?

No



* Notice when the constant is negative, the graph reflects over the x-axis.

Example #1

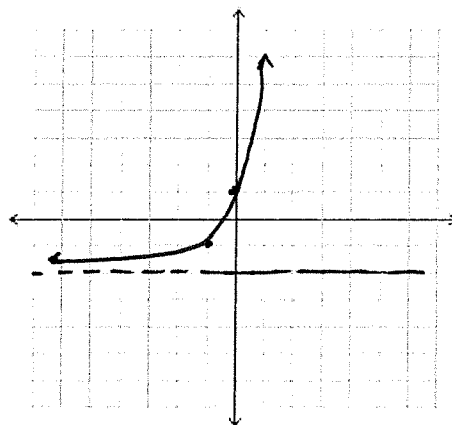
Graph the function $f(x) = 3^{x+1} - 2$

Domain: \mathbb{R}

Range: $y > -2$

Asymptotes: $y = -2$

Intercepts: y-int: $f(0) = 3^{0+1} - 2 = 3 - 2 = 1$



moves left 1
& down 2

Example #2

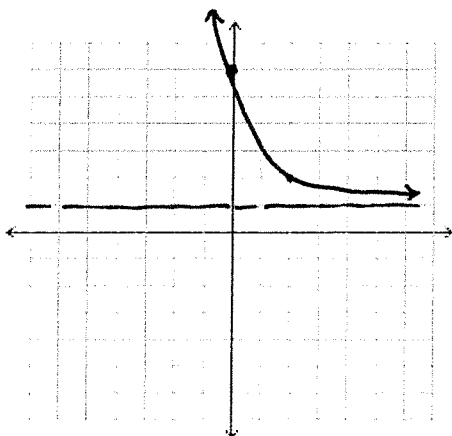
Graph the function $f(x) = \left(\frac{1}{2}\right)^{x-2} + 1$

Domain: \mathbb{R}

Range: $y > 1$

Asymptotes: $y = 1$

Intercepts: y-int: $f(0) = \frac{1}{2}^{0-2} + 1 = 5$



flips over y-axis
moves right 2
moves up 1

