# Precalculus Unit 2: 2.3 Notes
# Real Zeros of Polynomial Functions

1. **The Factor Theorem**: A polynomial $f(x)$ has a factor $(x-a)$ iff $f(a)=0$.

   EX: Show that $x=2$ is a solution to $x^3-7x+6=0$ and use the result to factor completely.

   EX: Show that $(x-2)$ and $(x+3)$ are factors of $f(x)=2x^4+7x^3-4x^2-27x-18$. Then factor completely.

2. **The Rational Zero Test**: If a polynomial has integer coefficients, and *p* and *q* are relatively prime, then every rational zero is of the form $\frac{p}{q}$ where p is a factor of the constant term and *q* is a factor of the leading coefficient.

   EX: List the possible rational solutions for the function *f* given by $f(x)=6x^3-x^2+9x+4$.

EX: Find the possible rational zeros of $f(x) = 2x^3 + 3x^2 - 8x + 3$.

3. **DesCartes Rule of Signs**: Let $f(x)$ be a polynomial with real coefficients
    a. The number of positive real zeros of *f* is either equal to the number of sign changes of $f(x)$ or less than that by an even integer.
    b. The number of negative real zeros of *f* is either equal to the number of sign changes of $f(-x)$ or less than that by an even integer.

EX: Find the number of real zeros for the function *f* given by $f(x) = 2x^3 + 3x^2 - 8x + 3$.

**4. Putting It All Together:**

Ex: Find all real zeros of the polynomial $f(x) = x^4 - x^3 - 29x^2 - x - 30$.

Ex: Find all real zeros of the polynomial $f(x) = 8x^4 - 14x^3 - 71x^2 - 10x + 24$.