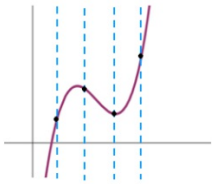
1.3 - Graphs of Functions

Vertical Line Test:

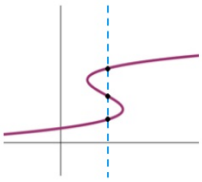
Vertical Line Test

A graph represents a function if there are no vertical lines that intersect the graph at more than one point.



Is a Function

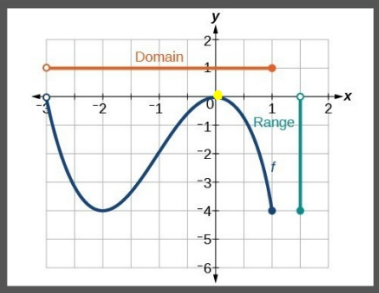
No vertical line will cross the graph more than once.



NOT a Function

There is a vertical line that crosses the graph more than once.

Domain/Range from a Graph:

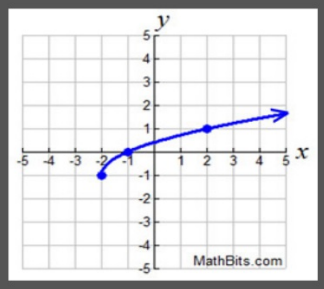


Domain:

$-3 < x \leq 1$

Range:

$-4 \leq y \leq 0$



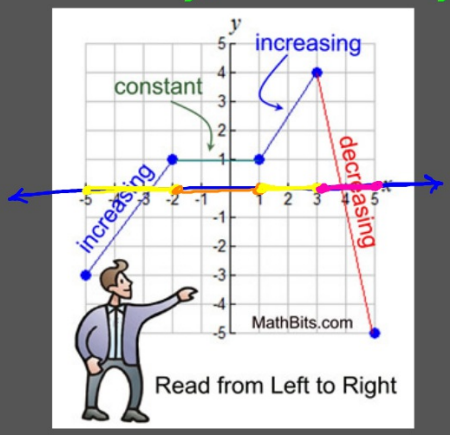
Domain:

$x \geq -2$

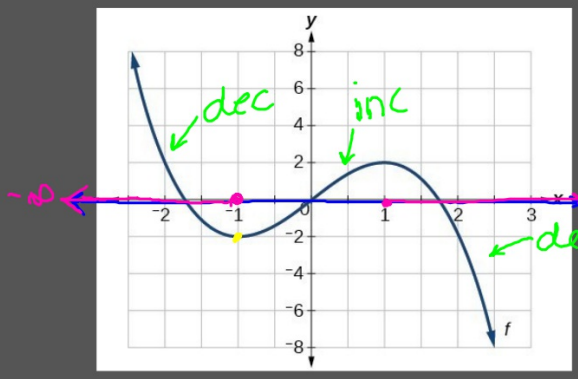
Range:

$-1 \leq y$

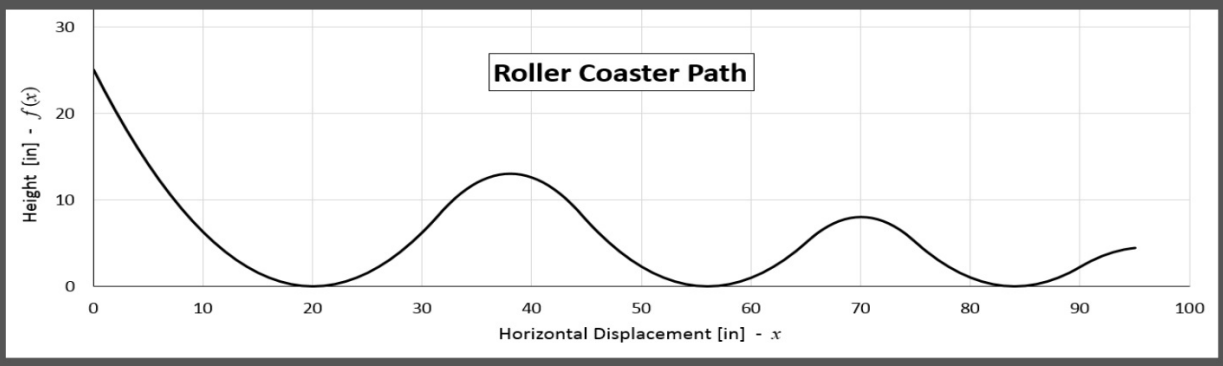
Increasing, Decreasing, Constant:



Increasing: $(-5,-2) \cup (1,3)$
Decreasing: $(3,5)$
Constant: $(-2,1)$



Increasing: $(-1,1)$
Decreasing: $(-\infty,-1) \cup (1,\infty)$
Constant:

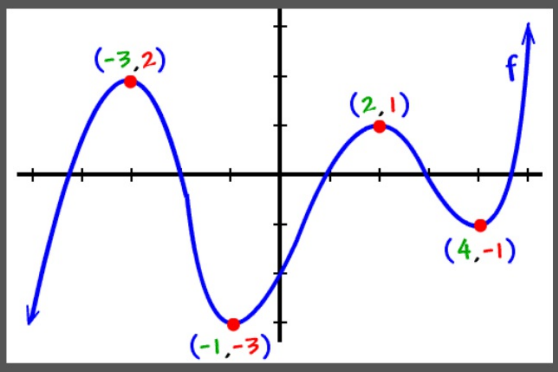


Increasing: $(20,38) \cup (54,70) \cup (83,96)$

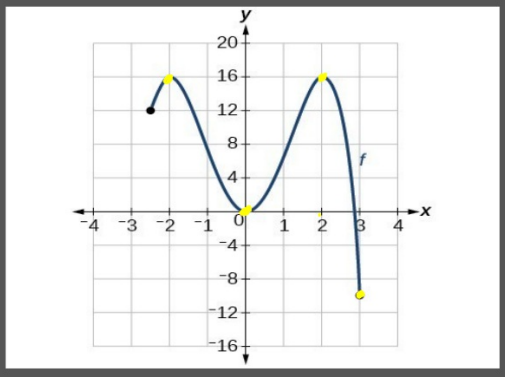
Decreasing: $(0,20) \cup (39,57) \cup (70,84)$

Constant:

Relative Maximum and Minimum Values:



Maxima: $(-3, 2), (2, 1)$
Minima: $(-1, -3), (4, -1)$



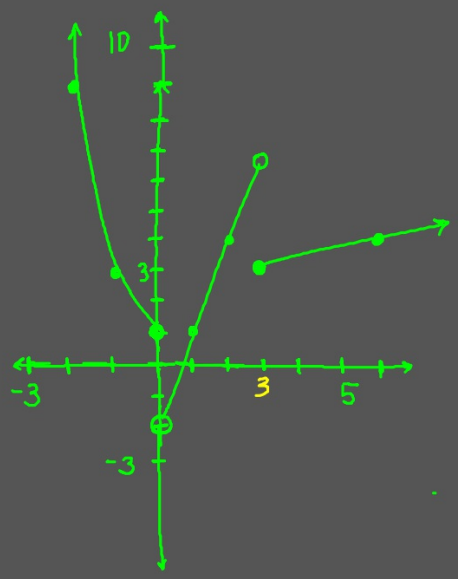
maxima:
Minima:

Relative Maxima and Minima Using Technology:

$f(x) = x^3 + 3x^2 - 2x + 1$

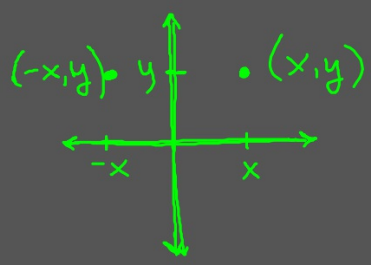
Graphing Piece-wise Functions:

$$f(x) = \begin{cases} 2x^2 + 1 & x \leq 0 \\ 3x - 2 & 0 < x < 3 \\ \frac{1}{3}x + 2 & x \geq 3 \end{cases}$$



Even/Odd Functions: Symmetry

Even → y-axis symmetry



replace x with $-x$

$$f(x) = 3x^4 - 2x^2 - 5$$

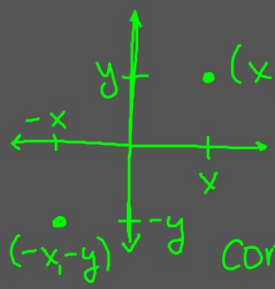
$$f(-x) = 3(-x)^4 - 2(-x)^2 - 5$$

$$= 3x^4 - 2x^2 - 5$$

compare
same ⇒
even

Even/Odd Functions:

Odd → rotational symmetry 180° origin



$y = 3x^3 - 2x + 2$

compare $-y = 3(-x)^3 - 2(-x) + 2$

$\frac{-y}{-1} = \frac{-3x^3}{-1} + \frac{2x}{-1} + \frac{2}{-1}$

$y = 3x^3 - 2x - 2$

∴ Same ⇒ odd